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Solution to Midterm, Fall 2014
Probability and Statistics for Engineers.

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1 Short Answer Questions

1.1. We write :

$$\begin{aligned}
\mu &= \int_0^1 xf(x)dx \\
&= 6\int_0^1 (x^2 - x^3)dx \\
&= 6(\frac{1}{3} - \frac{1}{4}) \\
&= \frac{1}{2}.
\end{aligned}$$

To calculate the variance, we note :

$$\begin{aligned}
\mathbb{E}[X^2] &= \int_0^1 x^2 f(x)dx \\
&= 6\int_0^1 (x^3 - x^4)dx \\
&= 6(\frac{1}{4} - \frac{1}{5}) \\
&= \frac{3}{10}.
\end{aligned}$$

We then have

$$
\begin{aligned}
\sigma^2 &= \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\
&= \frac{3}{10} - \frac{1}{4} \\
&= \frac{1}{20}.
\end{aligned}
$$

1.2. We write

$$
\begin{aligned}
\mathbb{P}[0.5 < X < 1] &= \int_{0.5}^{1} f(x)dx \\
&= 6\int_{0.5}^{1} (x - x^2)dx \\
&= \frac{1}{2}.
\end{aligned}
$$

2. Let A, B, C, D be the events that components 1,2,3 and 4 work, respectively. Let W be the event that the circuit works. Then

$$
W = A \cap ((B \cap C) \cup D),
$$

and so

$$
\begin{aligned}
\mathbb{P}[W] &= \mathbb{P}[A \cap ((B \cap C) \cup D)] \\
&= \mathbb{P}[A](\mathbb{P}[B \cap C] + \mathbb{P}[D] - \mathbb{P}[B \cap C \cap D]) \\
&= \mathbb{P}[A](\mathbb{P}[B]\mathbb{P}[C] + \mathbb{P}[D] - \mathbb{P}[B]\mathbb{P}[C]\mathbb{P}[D]) \\
&= \frac{9}{10}(\frac{16}{25} + \frac{4}{10} - \frac{64}{250}) \\
&= \frac{9}{10}\frac{98}{125} \\
&= 0.7056.
\end{aligned}
$$

2 Multiple Choice

1. Let A denote the event that somebody is accident-prone and let B denote the event that they have an accident.

$$\begin{aligned}\mathbb{P}[A|B] &= \frac{\mathbb{P}[B|A]\mathbb{P}[A]}{\mathbb{P}[B|A]\mathbb{P}[A] + \mathbb{P}[B|A^c]\mathbb{P}[A^c]} \\ &= \frac{5}{9}.\end{aligned}$$

2. Let X denote the number of samples containing a sample particle.

$$\mathbb{P}[X \geq 1] = 1 - \mathbb{P}[X = 0] = 1 - (0.9)^{20} \approx 0.8784.$$

3. If they were mutually exclusive, we would have

$$\begin{aligned}\mathbb{P}[A \cup B] &= \mathbb{P}[A] + \mathbb{P}[B] \\ &= 1.15 \\ &> 1.\end{aligned}$$

But the probability of an event is at most 1. Thus, they are not mutually exclusive.

4. Recognizing the distribution, we have

$$p = \frac{\binom{7}{3}\binom{23}{9}}{\binom{30}{12}} \approx 0.331.$$

5. There are 32 who own a car and live on campus, and $340 + 32 = 372$ students who live on campus. Thus, the answer is $\frac{32}{372} = \frac{8}{93}$.

6. Note the distribution is discrete, with all mass at 0,2,3, and 4. Thus,

$$\begin{aligned}\mathbb{P}[1 \leq X \leq 3] &= \mathbb{P}[X \in \{1,3\}] \\ &= (0.42 - 0.37) + (0.37 - 0.12) \\ &= 0.3.\end{aligned}$$

7. The number of calls is Poisson with rate $\lambda = 8\frac{1}{2} = 4$. Write this as X. We have

$$\begin{aligned}\mathbb{P}[X \leq 3] &= \mathbb{P}[X = 0] + \mathbb{P}[X = 1] + \mathbb{P}[X = 2] + \mathbb{P}[X = 3] \\ &\approx 0.433.\end{aligned}$$

8. Let S be the event that a course is taken, and let A be the event that there is an accident. We have

$$\begin{aligned}\mathbb{P}[A] &= \mathbb{P}[A|S]\mathbb{P}[S] + \mathbb{P}[A|S^c]\mathbb{P}[S^c] \\ &= (0.005)(0.3) + (0.018)(0.7) \\ &\approx 0.0141.\end{aligned}$$